Computer Logic (years 9, 10, 11)

Basic Logic Operators

The basic logical operators are AND, OR, NOT, NAND, NOR and XOR.

X	Y	X AND Y
T	T	T
T	F	F
F	T	F
F	F	F

T stands for True and F for False. In the computer environment T and F are changed respectively to 1 and 0.

X	Y	X AND Y
1	1	1
1	0	0
0	1	0
0	0	0

The AND operator is also represented by a dot and this dot is sometimes left out too for example X AND Y can be represented by X.Y or even XY.

X	Y	X OR Y
1	1	1
1	0	1
0	1	1
0	0	0

The OR operator is also represented by a + for example X OR Y can be represented by X + Y.

X	NOT X
1	0
0	1

NOT X can be written as X' or $\overline{X}$.

XOR stands for Exclusive OR

X	Y	X XOR Y
1	1	0
1	0	1
0	1	1
0	0	0

NAND stands for Not AND

X	Y	X NAND Y
1	1	0
1	0	1
0	1	1
0	0	1

NOR stands for Not OR

X	Y	X NOR Y
1	1	0
1	0	0
0	1	0
0	0	1

XNOR stands for Not Exclusive OR

X	Y	X XNOR Y
1	1	1
1	0	0
0	1	0
0	0	1

Logical (Boolean) Expressions

$$A + \overline{B}$$

$$\overline{AB} + C$$

$$A(B + \overline{C})$$

$$(\overline{A} + B)(A + \overline{C})$$

Precedence of Operators

When more than one logical operator is used in a statement, NOT is evaluated first, then AND, and finally OR. As in arithmetic the brackets are worked out first. Example: Let A=1, B=0 and C=1.

$$A + \overline{B}$$

evaluates to A OR NOT B =

1 OR NOT 0 =

1 OR 1 =

1

$$\overline{AB} + C$$

evaluates to NOT (A AND B) OR C =

NOT (1 AND 0) OR 1 =

NOT (0) OR 1 =

1 OR 1 =

1

$$A(B + \overline{C})$$

evaluates to A AND (B OR NOT C) =

1 AND (0 OR NOT 1) =

1 AND (0 OR 0) =

1 AND 0 =

0

Exercise 1: Evaluate the following Boolean expressions given that A=0, B=1 and C=1.

a) $\overline{A} + C$

b) $A + \overline{B}.\overline{C}$

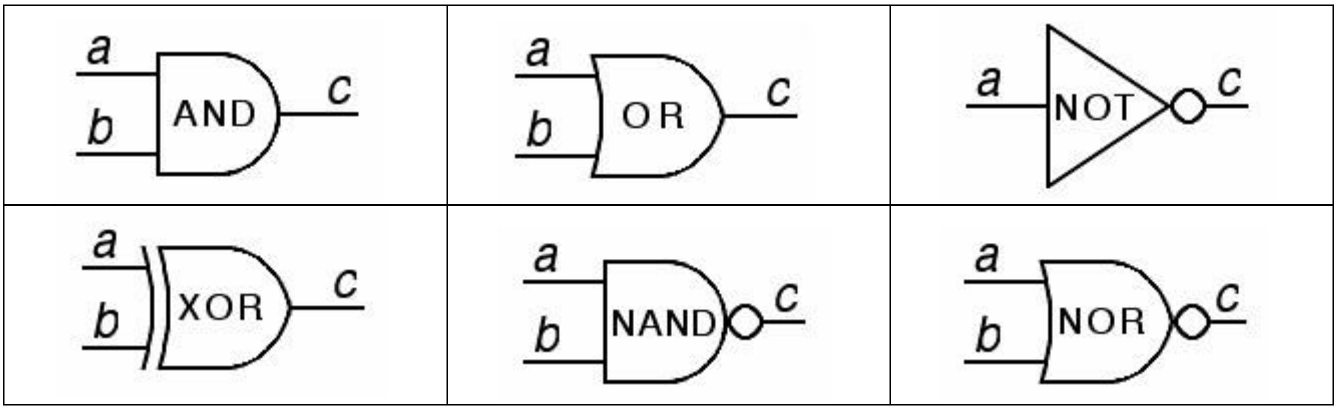
c) $A(\overline{B} + \overline{AC})$

d) $(A + \overline{B})(A + \overline{C})$

Logic Gates

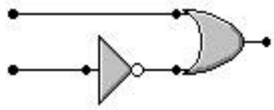
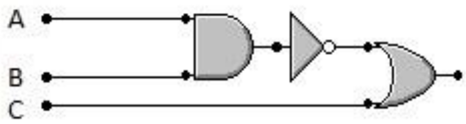
A logic gate is an elementary building block of a digital circuit. Most logic gates have two inputs and one output. At any given moment, every terminal is in one of the two binary conditions low (0) or high (1), implemented by different voltage levels. In most logic gates, the low state is approximately zero volts (0 V), while the high state is approximately five volts positive (+5 V).

The following table shows the most common logic gates.



Logic Circuits

Boolean expressions can be expressed as logic circuits. Examples are shown hereunder:

Boolean Expression	Logic Circuit
$A + \overline{B}$	
$\overline{AB} + C$	

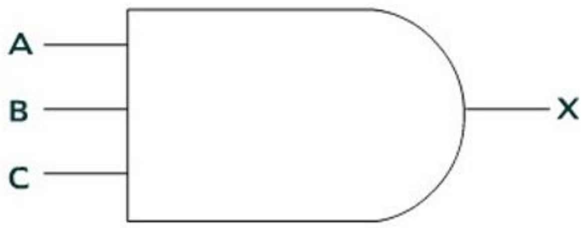
Exercise 2:

1. Draw the logic circuits of the Boolean expressions found in exercise 1.
2. Draw the truth tables of the same expressions.
3. Prove the following identities by showing that the truth table of the left-hand expression is identical to the truth table of the right-hand expression.

   a. $AB + A\overline{B} \equiv A + \overline{B}$

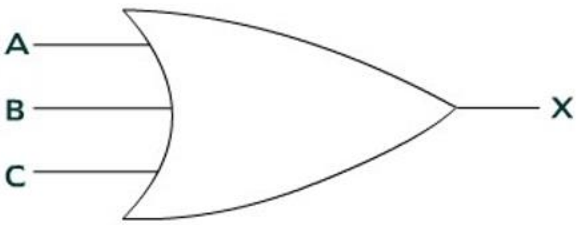
   b. $(P + \overline{Q})(P + Q) \equiv P$

Gates with Three inputs



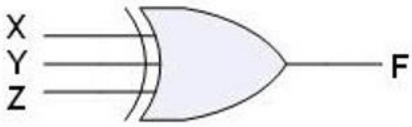
$$X = A \cdot B \cdot C$$

Inputs			Output
C	*B*	*A*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



$$X = A + B + C$$

Inputs			Output
C	*B*	*A*	*X*
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



3-input XOR gate			
A	**B**	**C**	**Output**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1